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SCM-based Geometry Smoothing for the SBFEM on Quadtree Meshes

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ABSTRACT

In engineering practice the finite element method (FEM) is used to solve a wide class of problems commonly given as partial differential equations (PDEs). The FEM, however, requires a subdivision of the computational domain into non-overlapping and geometry-conforming subdomains, the so called finite elements. Considering problems involving localized effects, such as steep gradient or discontinuities a fine spatial resolution is necessary to obtain accurate results. Therefore, adaptive mesh refinement methods are often applied in those cases.
One idea is to deploy a quadtree-discretization of the domain. This approach is relatively easy to implement and allows for an automatic mesh generation process but also suffers from the existence of hanging nodes. A possible remedy is to exploit the advantages of a semi-analytical approach known as the scaled boundary finite element method (SBFEM). Here, the governing partial differential equations are reduced to a set of ordinary differential equations (ODEs) by only discretizing the boundary of the domain in a FE-sense. Therefore, each edge of a quadtree mesh can be discretized with an arbitrary number of one-dimensional finite elements featuring arbitrary polynomial degrees. Consequently, hanging nodes are not an issue in the SBFEM [5, 3].
Despite the mentioned advantages the approximation of curved boundaries requires additional treatment. Although quadtree cells intersecting with the physical boundary of the domain are further refined it is impossible to generate a smooth surface when only relying on a pure quadtree decomposition. Due to the staircase-nature of the geometry approximation unrealistically high stresses may be computed [5]. In order to accurately describe the boundaries of the structure there are three options: (i) quadtree cells are trimmed into arbitrary (star-shaped) polygons [5]; (ii) nodes are repositioned to match the boundary (*r*-FEM) [4]; (iii) a fictitious domain approach is deployed [2]. In the current contribution the third approach is chosen. Therefore, it is proposed to couple the quadtree-based SBFEM with the spectral cell method (SCM). The SCM is the combination of a high order FEM with a fictitious domain method [1]. As in the SBFEM the shape functions are commonly defined by Lagrangian interpolation polynomials based on a non-equidistant Gauß-Lobatto-Legendre (GLL) nodal distribution. Due to the identical shape function basis the coupling of both methods is straightforward and does not introduce any new problems. The actual geometry of the computational domain is taken into account during the quadrature of the weak form. The effort to create a geometry-conforming discretization is thus shifted to an advanced integration concept [2].

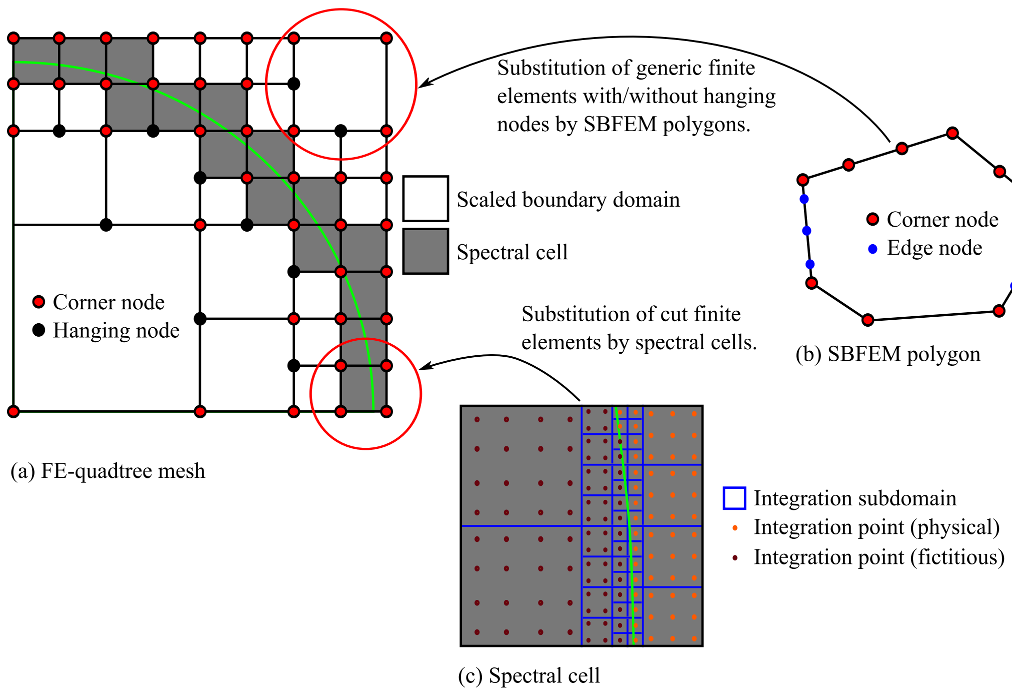

Figure 1: Quadtree-based scaled boundary spectral cell method: (a) Generic finite element quadtree mesh with hanging nodes; (b) Scaled boundary polygon with different numbers of line elements and polynomial degrees; (c) Cut finite cell with integration subdomains and quadrature points.

In the novel scheme, the SCM concept is only employed for the quadtree cells that are intersected by the physical boundary, cf. Fig. 1. In order to decrease the numerical effort all other cells are treated in a SBFEM-sense. The combination of both methods is labeled as SBSCM. The capabilities of the novel method are demonstrated by means of several numerical examples including static image-based analyses of complex geometries and the simulation of wave propagation phenomena in severely inhomogeneous domains.

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